# vGraph: A Generative Model for Joint Community Detection and Node Representation Learning

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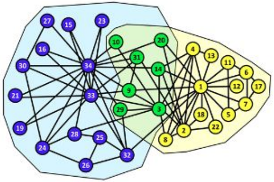
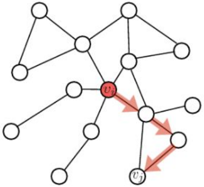
# Introduction

Graphs are a general and flexible data structure to encode complex relationships among objects. Examples of real-world graphs include social networks, airline networks, protein-protein interaction networks, and traffic networks. This paper focuses on two fundamental tasks of graph analysis: **community detection** and **node representation learning**, which capture the global and local structures of graphs, respectively.

# Motivation

(Non)Overlapping Community Detection




Node Representation Learning
- MF
- LINE
- DeepWalk
- Node2vec

# Motivation

(Overlapping) Community Detection

Community Preserving Network Embedding

Clustering (i.e. K-Means) Using Node Embeddings as feature

Node Representation Learning

- MF
- LINE
- DeepWalk
- Node2vec

# vGraph - probabilistic generative model

- Generative model with Variational Inference to solve community detection and node representation learning jointly.
- Scalable: $O(d * |E| * K)$.
  - $d$: embedding dimension
  - $|E|$: number of edges in the graph
  - $K$: number of communities

# Problem Definition

Given a graph $G(V, E)$ where $V$ and $E$ represent the set of vertices and edges respectively, we aim to jointly learn a node embedding $\phi_i \in \mathbb{R}^d$ and community affiliation $\mathcal{F}(\nu_i) \subseteq \{1, ..., K\}$ for each vertex $\nu_i$.

# Model

vGraph assumes that the edges $(w, c)$ is generated from the following stocastic process: for node $w$, we first draw a community assignment $z \sim p(z|w)$ representing the social context of $w$ during the generation process. Then, the linked neighbor $c$ is generated based on the assignment $z$ through $c \sim p(c|z)$. Formally, this process can be fomulated as:

$$p(c|w) = \sum_z p(c|z)p(z|w)$$

# Model

$$p(c|w) = \sum_z p(c|z)p(z|w)$$

vGraph parameterizes the distributions $p(z|w)$ and $p(c|z)$ by introducing three sets of embeddings: $\phi_i$, $\varphi_i$, and $\psi_j$. The prior distribution $p_{\phi,\psi}(z|w)$ and the node distribution conditioned on a community $p_{\psi,\varphi}(c|z)$ are parameterized by two softmax models:

$$p_{\phi,\psi}(z = j|w) = \frac{\exp(\phi_w^T \psi_j)}{\sum_{i=1}^{K} \exp(\phi_w^T \psi_i)}$$

$$p_{\psi,\varphi}(c|z = j) = \frac{\exp(\psi_j^T \varphi_c)}{\sum_{c\prime \in N} \exp(\psi_j^T \varphi_{c\prime})}$$

# Variational Inference

The goal is to maximize the log-likelihood of the observed edges

$$\sum_{(c,w)\in E} \log p_{\phi,\varphi,\psi}(c|w)$$

Directly optimizing this objective is intractable for large graphs, we instead optimize the following evidence lower bound

$$\mathcal{L} = E_{z\sim q(z|c,w)}(\log p_{\psi,\varphi}(c|z)) - KL(q(z|c,w)\|p_{\phi,\psi}(z|w))$$

where $q(z|c,w)$ is a variational distribution that approximates the true posterior distribution $p(z|c,w)$, and $KL(\cdot\|\cdot)$ represents the Kullback-Leibler divergence between two distributions.

# Variational Inference

$$\mathcal{L} = E_{z\sim q(z|c,w)}(\log p_{\psi,\varphi}(c|z)) - KL(q(z|c,w)\|p_{\phi,\psi}(z|w))$$

$$p_{\phi,\psi}(z=j|w) = \frac{\exp(\phi_w^T\psi_j)}{\sum_{i=1}^{K}\exp(\phi_w^T\psi_i)}$$

$$p_{\psi,\varphi}(c|z=j) = \frac{\exp(\psi_j^T\varphi_c)}{\sum_{c'\in N}\exp(\psi_j^T\varphi_{c'})}$$

Specifically, we parameterize the variational distribution using a neural network as follows ($\odot$ denotes element-wise multiplication):

$$q_{\phi,\psi}(z=j|c,w) = \frac{\exp((\phi_w\odot\phi_c)^T\psi_j)}{\sum_{i=1}^{K}\exp((\phi_w\odot\phi_c)^T\psi_i)}$$

# Straight-through Gumbel-Softmax estimator

It refactors the sampling operation into a deterministic function.

$$z = \underset{i}{argmax}\{G_i + log(\pi_i)\}$$

where $G_i \sim Gumbel(0, 1)$. To pass gradients back, Straight-through estimator is used, which is basically using softmax to approximate the argmax operation during back propagation.

# Community-smoothness Regularization

vGraph add the following regularization term to ensure that connected nodes tend to be in the same community:

$$\mathcal{L}_{reg} = \lambda \sum_{(w,c)\in E} \alpha_{w,c} \cdot d(p(z|c), p(z|w))$$

where $\lambda$ is a tunable hyperparameter, $d(\cdot,\cdot)$ is the distance measure (squared difference in the experiment). $\alpha_{w,c}$ is given by:

$$\alpha_{w,c} = \frac{|N(w) \cap N(c)|}{|N(w) \cup N(c)|}$$

where $N(w)$ is the set of neighbors of $w$. The intuition behind this is that $\alpha_{w,c}$ serves as a similarity measure and the Jaccard's coefficient is used for this metric.

# Experiment Setup

- 20 standard graph datasets.
- 3 tasks: overlaping community detection, non-overlaping community detection, and vertex classification.

# Overlapping community detection

Table 2: Evaluation (in terms of F1-Score and Jaccard Similarity) on networks with overlapping ground-truth communities. NA means the task is not completed in 24 hours. In order to evaluate the effectiveness of smoothness regularization, we show the result of our model with (vGraph+) and without the regularization.

| | F1-score | | | | | | Jaccard | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dataset | Bigclam | CESNA | Circles | SVI | vGraph | vGraph+ | Bigclam | CESNA | Circles | SVI | vGraph | vGraph+ |
| facebook0 | **0.2948** | 0.2806 | 0.2860 | 0.2810 | 0.2440 | 0.2606 | **0.1846** | 0.1725 | 0.1862 | 0.1760 | 0.1458 | 0.1594 |
| facebook107 | **0.3928** | 0.3733 | 0.2467 | 0.2689 | 0.2817 | 0.3178 | **0.2752** | 0.2695 | 0.1547 | 0.1719 | 0.1827 | 0.2170 |
| facebook1684 | 0.5041 | **0.5121** | 0.2894 | 0.3591 | 0.4232 | 0.4379 | 0.3801 | **0.3871** | 0.1871 | 0.2467 | 0.2917 | 0.3272 |
| facebook1912 | 0.3493 | 0.3474 | 0.2617 | 0.2804 | 0.2579 | **0.3750** | 0.2412 | 0.2394 | 0.1672 | 0.2010 | 0.1855 | **0.2796** |
| facebook3437 | 0.1986 | 0.2009 | 0.1009 | 0.1544 | 0.2087 | **0.2267** | 0.1148 | 0.1165 | 0.0545 | 0.0902 | 0.1201 | **0.1328** |
| facebook348 | 0.4964 | 0.5375 | 0.5175 | 0.4607 | **0.5539** | 0.5314 | 0.3586 | 0.4001 | 0.3927 | 0.3360 | **0.4099** | 0.4050 |
| facebook3980 | 0.3274 | 0.3574 | 0.3203 | NA | **0.4450** | 0.4150 | 0.2426 | 0.2645 | 0.2097 | NA | **0.3376** | 0.2933 |
| facebook414 | 0.5886 | 0.6007 | 0.4843 | 0.3893 | 0.6471 | **0.6693** | 0.4713 | 0.4732 | 0.3418 | 0.2931 | 0.5184 | **0.5587** |
| facebook686 | 0.3825 | 0.3900 | 0.5036 | 0.4639 | 0.4775 | **0.5379** | 0.2504 | 0.2534 | 0.3615 | 0.3394 | 0.3272 | **0.3856** |
| facebook698 | 0.5423 | 0.5865 | 0.3515 | 0.4031 | 0.5396 | **0.5950** | 0.4192 | 0.4588 | 0.2255 | 0.3002 | 0.4356 | **0.4771** |
| Youtube | 0.4370 | 0.3840 | 0.3600 | 0.4140 | 0.5070 | **0.5220** | 0.2929 | 0.2416 | 0.2207 | 0.2867 | 0.3434 | **0.3480** |
| Amazon | 0.4640 | 0.4680 | **0.5330** | 0.4730 | **0.5330** | 0.5320 | 0.3505 | 0.3502 | 0.3671 | 0.3643 | 0.3689 | **0.3693** |
| Dblp | 0.2360 | 0.3590 | NA | NA | 0.3930 | **0.3990** | 0.1384 | 0.2226 | NA | NA | 0.2501 | **0.2505** |
| Coauthor-CS | 0.3830 | 0.4200 | NA | 0.4070 | 0.4980 | **0.5020** | 0.2409 | 0.2682 | NA | 0.2972 | **0.3517** | 0.3432 |

# Non-overlapping community detection

Table 3: Evaluation (in terms of NMI and Modularity) on networks with non-overlapping ground-truth communities.

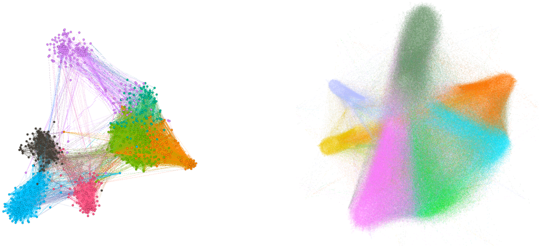
| | NMI | | | | | | Modularity | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dataset | MF | deepwalk | LINE | node2vec | ComE | vGraph | MF | deepwalk | LINE | node2vec | ComE | vGraph |
| cornell | 0.0632 | 0.0789 | 0.0697 | 0.0712 | 0.0732 | **0.0803** | 0.4220 | 0.4055 | 0.2372 | 0.4573 | 0.5748 | **0.5792** |
| texas | 0.0562 | 0.0684 | **0.1289** | 0.0655 | 0.0772 | 0.0809 | 0.2835 | 0.3443 | 0.1921 | 0.3926 | **0.4856** | 0.4636 |
| washington | 0.0599 | 0.0752 | **0.0910** | 0.0538 | 0.0504 | 0.0649 | 0.3679 | 0.1841 | 0.1655 | 0.4311 | 0.4862 | **0.5169** |
| wisconsin | 0.0530 | 0.0759 | 0.0680 | 0.0749 | 0.0689 | **0.0852** | 0.3892 | 0.3384 | 0.1651 | 0.5338 | 0.5500 | **0.5706** |
| cora | 0.2673 | 0.3387 | 0.2202 | 0.3157 | **0.3660** | 0.3445 | 0.6711 | 0.6398 | 0.4832 | 0.5392 | 0.7010 | **0.7358** |
| citeseer | 0.0552 | 0.1190 | 0.0340 | 0.1592 | **0.2499** | 0.1030 | 0.6963 | 0.6819 | 0.4014 | 0.4657 | 0.7324 | **0.7711** |

# Vertex classification

Table 4: Results of node classification on 6 datasets.

| | Macro-F1 | | | | | | Micro-F1 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Datasets | MF | DeepWalk | LINE | Node2Vec | ComE | vGraph | MF | DeepWalk | LINE | Node2Vec | ComE | vGraph |
| Cornell | 13.05 | 22.69 | 21.78 | 20.70 | 19.86 | **29.76** | 15.25 | 33.05 | 23.73 | 24.58 | 25.42 | **37.29** |
| Texas | 8.74 | 21.32 | 16.33 | 14.95 | 15.46 | **26.00** | 14.03 | 40.35 | 27.19 | 25.44 | 33.33 | **47.37** |
| Washington | 15.88 | 18.45 | 13.99 | 21.23 | 15.80 | **30.36** | 15.94 | 34.06 | 25.36 | 28.99 | 33.33 | **34.78** |
| Wisconsin | 14.77 | 23.44 | 19.06 | 18.47 | 14.63 | **29.91** | 18.75 | **38.75** | 28.12 | 25.00 | 32.50 | 35.00 |
| Cora | 11.29 | 13.21 | 11.86 | 10.52 | 12.88 | **16.23** | 12.79 | 22.32 | 14.59 | 27.74 | **28.04** | 24.35 |
| Citeseer | 14.59 | 16.17 | 15.99 | 16.68 | 12.88 | **17.88** | 15.79 | 19.01 | 16.80 | **20.82** | 19.42 | 20.42 |

# Visualization



(a) (b)

Figure 2: In panel (a) we visualize the result on the facebook107 dataset using vGraph. In panel (b) we visualize the result on Dblp-full dataset using vGraph. The coordinates of the nodes are determined by t-SNE of the node embeddings.

# Conclusion

This paper presented a probabilistic method for jointly solving community detection and node representation learning. Experiments show that it performs well for both tasks.

Thank you!